

Mathematical Modeling and Volumetric Estimation of a Three Phase Fluid Flow in a Porous Media Using Eclipse 100. A Case Study of a Niger Delta Reservoir

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Abstract

A three-dimensional and multiphase mathematical model was developed to estimate the volume of oil, gas, water in the reservoir. The mathematical model equations were derived based on the principle of conservation of mass. The numerical solution of these model equations involved the use of finite difference method of approximations for the discretization of the partial differential equations. The simulated results which were obtained using a dynamic simulator known as ECLIPSE 100, showed the volume of oil, gas and water to be 51220873m^3 , $1.722041\text{E} + 8\text{m}^3$ and $247533\text{E} + 10\text{m}^3$, respectively. This result showed that the reservoir is a gas reservoir since it is producing more of gas.

Key Words: Mathematical model, Three-phase fluid flow, and porous media

1.0 Introduction

Flow in porous media is very complex phenomenon and as such cannot be describe as explicitly as flow through pipes or conducts. It is rather easy to measure the length and diameter of a pipe and compute its flow capacity as a function of pressure. In porous media, however flow is different in that there is no clear-cut flow paths which lend themselves to measurement.

The analysis of fluid flow in porous media has evolved throughout the years along two fronts- the experimental and analytical. Physicists, engineers, hydrologist and the like have examined experimentally the behavior of various fluids as they flow through porous media ranging from sand packs to fused Pyrex glass, on the basis of their analyses, they have attempted to formulate laws and correlations that can then be utilized to make analytical predictions for similar systems.

However, there are mathematical models which have been designed to describe the flow behavior of the reservoir fluids. Porous medium is substance that contains pores or spaces between solid materials through which liquid or gas can pass (Corey, 1999). Pores are either connected or inter-connected. A fluid can only flow through a porous medium only if at least some of the pores are inter-connected. The inter-connected pore space is termed the effective pore space, while the whole of the pore space is termed the total pore space. Examples of a porous medium are an aquifer from which ground water is pumped, reservoirs which yield oil and /or gas (Bear J. and Bachmat Y., 1991). The voids within a porous medium can further be classified according to their sizes. Three main classifications are possible based on the behavior³ of fluid within the void space. In the smallest void spaces molecular forces between

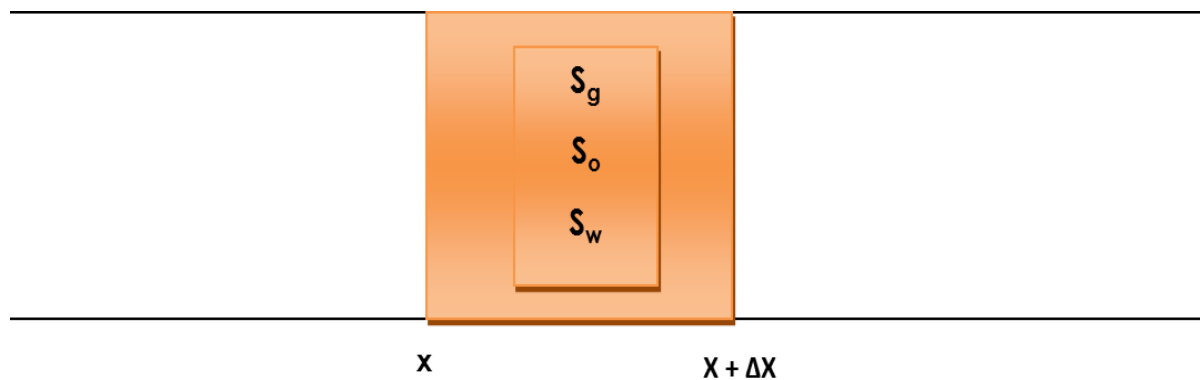
the molecule of the solid and those of the liquid are significant. These tiniest void spaces are termed molecular interstices. In the largest void spaces the motion of a fluid is only partially determined by the walls of the void: These largest spaces are referred to as caverns. Those spaces which are intermediate in size between molecular interstice and caverns are termed pores. (Collins, 1961).

The objective of this work is to develop a multidirectional mathematical model capable of predicting the performance of a multiphase hydrocarbon system comprising of gas, oil, and water. The scrubbing effect and the miscibility of the gas in oil phase are considered.

2.0 Mathematical Modeling of Fluid Flow Dynamics in Porous Media.

a. Continuity Equations for Multiphase Flow System

When there are more than one fluid in a system, there exist complex interactions between the various components of the fluid systems. The Continuity Equation must be written for each of the fluids. In other words, each fluid in the system must be conserved irrespective of the form or phase in which the fluid exists. It is very pertinent to note that all physical processes take place within certain confines of space and time. Considering flow in the linear reservoir below:



Multiphase One Dimensional system.

The Mass Conservation Equation for fluid *i* in the shaded control volume is given by:

$$\text{Rate of Mass In} + \text{Rate of Mass Injection} - \text{Rate of Mass Out} - \text{Rate of Mass Production} = \text{Rate of Mass Storage.} \quad (1)$$

Let us assume that fluid *i* is being injected and produced at volumetric rate per unit volume of q_{inisc} and q_{outsc} respectively measured at standard conditions.

If the fluid has a saturation of S_i in the system, The Conservation Equation becomes:

$$-\nabla(\rho_i u_i) = \frac{\partial}{\partial t} [\rho_i \phi S_i] + \rho_{isc} (q_{outsc} - q_{inisc}) \quad (2)$$

This Equation can be written for gas, oil and water as seen below:

For gas:

$$-\nabla(\rho_g u_g) = \frac{\partial}{\partial t} [\rho_g \phi S_g] + \rho_{gsc} (q_{outsc} - q_{inisc}) \quad (3)$$

For Oil:

$$-\nabla(\rho_o u_o) = \frac{\partial}{\partial t} [\rho_o \phi S_o] + \rho_{osc}(q_{outsc} - q_{insec}) \quad (4)$$

For Water:

$$-\nabla(\rho_w u_w) = \frac{\partial}{\partial t} [\rho_w \phi S_w] + \rho_{wsc}(q_{outsc} - q_{insec}) \quad (5)$$

b. The Transport Rate Equation for Multiphase Fluid System

The general form of the Darcy Velocity for a fluid i is:

$$u_i = - \frac{kk_{ri}}{\mu_i} (\nabla p_i - \gamma_i \Delta Z) \quad (6)$$

This equation implies that:

For Gas:

$$u_g = - \frac{kk_{rg}}{\mu_g} (\nabla p_g - \gamma_g \Delta Z) \quad (7)$$

For Oil:

$$u_o = - \frac{kk_{ro}}{\mu_o} (\nabla p_o - \gamma_o \Delta Z) \quad (8)$$

For Water:

$$u_w = - \frac{kk_{rw}}{\mu_w} (\nabla p_w - \gamma_w \Delta Z) \quad (9)$$

The pressure gradients in Eqns. 7 – 9 refer to the pressure gradients in each phase. The pressures in each phase are not the same because of capillary pressure.

In Equation 2, the term $\rho_i u_i$ is the mass flow rate of fluid i per unit area open to flow. For this system under consideration, the term is defined as follows:

For Gas:

$\rho_g u_g$ = total mass flow rate of gas per unit area = mass flow rate of free gas per unit area + mass flow rate of dissolved gas per unit area, i.e.

$$\rho_g u_g = \rho_g u_g + \rho_{gstc} R_s \frac{u_o}{B_o} \quad (10)$$

Also, $\rho_g \phi S_g$ = total mass of gas in control volume per unit volume = mass of gas in the gas phase in the control volume + mass of gas in the oil phase in the control volume, i.e.

$$\rho_g \phi S_g = \phi \left[\rho_g S_g + \frac{S_o}{B_o} R_s \rho_{gstc} \right] \quad (11)$$

The gas density in terms of the FVF at reservoir condition is given as:

$$-\nabla \left\{ \frac{\rho_{gsc}}{B_g} u_g + \rho_{gsc} R_s \frac{u_o}{B_o} \right\} = \frac{\partial}{\partial t} \left\{ \phi \left[\frac{\rho_{gsc}}{B_g} S_g + \frac{S_o}{B_o} R_s \rho_{gsc} \right] \right\} + \rho_{gsc} (q_{goutsc} - q_{ginsc}) \quad (12)$$

Equation (12) can be simplified to give:

$$-\nabla \left\{ \frac{u_g}{B_g} + R_s \frac{u_o}{B_o} \right\} = \frac{\partial}{\partial t} \left\{ \phi \left[\frac{S_g}{B_g} + \frac{S_o}{B_o} R_s \right] \right\} + q_{gsc} \quad (13)$$

Substituting Eqns. 7 and 8 in Eqn. 12, we have:

$$\nabla \left[\frac{kk_r}{B_g \mu_g} (\nabla p_g - \gamma_g \nabla Z) + \frac{R_s kk_{ro}}{B_o} (\nabla p_o - \gamma_o \nabla Z) \right] = \frac{\partial}{\partial t} \left\{ \phi \left[\frac{S_g}{B_g} + R_s \frac{S_o}{B_o} \right] \right\} + q_{gsc} \quad (14)$$

Following the same reasoning, the equations for oil and water stand out respectively as:

For Oil:

$\rho_o u_o$ is the total mass flow rate of oil per unit area, also, $\rho_o \phi S_o$ is total mass of oil in control volume per unit volume. Substituting the total mass of oil in control volume per unit volume and the total mass flow rate of oil per unit area in the Continuity Equation gives:

$$-\nabla \left(\frac{u_o}{B_o} \right) = \frac{\partial}{\partial t} \left(\frac{\phi S_o}{B_o} \right) + q_{osc} \quad (15)$$

where $q_{osc} = q_{oil\ outsc} - q_{oil\ insc}$

Substituting for the fluid velocity gives:

$$\nabla \left\{ \frac{kk_{ro}}{B_o \mu_o} (\nabla p_o - \gamma_o \nabla Z) \right\} = \frac{\partial}{\partial t} \left(\frac{\phi S_o}{B_o} \right) + q_{osc} \quad (16)$$

For water:

The Flow Equation for water is given as:

$$-\nabla \left(\frac{u_w}{B_w} \right) = \frac{\partial}{\partial t} \left(\frac{\phi S_w}{B_w} \right) + q_{wsc} \quad (17)$$

where $q_{wsc} = q_{water\ outsc} - q_{water\ insc}$

Substituting for the fluid velocity, Eq. 17 becomes:

$$\nabla \left\{ \frac{kk_{rw}}{B_w \mu_w} (\nabla p_w - \gamma_w \nabla Z) \right\} = \frac{\partial}{\partial t} \left(\frac{\phi S_w}{B_w} \right) + q_{wsc} \quad (18)$$

Where $q_{isc} = q_{ioutsc} - q_{iinsc}$

The assumptions made in arriving at these equations are:

- i. Darcy's law is applied
- ii. Constant viscosity and density
- iii. Permeability is anisotropic in space and time
- iv. The porous media is isothermal
- v. The porous media is heterogeneous
- vi. Gravity effects is negligible
- vii. Formation volume factor is unity

Volumetric Estimation of Oil and Gas

Volumetric estimates of original oil in place and original gas in place are based on a geological model that geometrically describes the volume of hydrocarbons in the reservoir. However, due mainly to gas evolving from the oil as pressure and temperature are decreased, oil at the surface occupies less space than it does in the subsurface. Conversely, gas at the surface occupies more space than it does in the subsurface because of expansion. This necessitates correcting subsurface volumes to standard units of volume measured at surface conditions using the following equations;

$$N = \frac{Ah\phi(1-s_w)}{B_{oi}} \quad (19)$$

$$G = \frac{Ah\phi(1-s_w)}{B_{gi}} \quad (20)$$

Resolving equations (14), (16) and (18) and substituting them in equations (19) and (20) will yield the volumetric estimate of the three phase fluids in the reservoir. However, in this work an eclipse simulation was done to estimate the volume of oil, water and gas using the parameters given above.

Results and Discussion

Table 1: Reservoir parameters

1	Porosity (ϕ)	0.35
2	Permeability: k_x , k_y , k_z ,	1D, 1D, 0.2D
3	Reference Pressure: P^0	400 bar
4	Rock compressibility: c	4.0e-5 bar
5	Oil Density: ρ_o	749.389 kg/m ³
6	Water density: ρ_w	1000kg/m ³
7	Gas density: ρ_g	1.11242kg/m ³
8	Datum depth	3000m
9	Pressure datum depth	331.65 bar
10	Water-Oil Contact (WOC) depth	3085 m
11	Oil-water Capillary Pressure at OWC	0 bar
12	Gas-Oil Contact (GOC) depth	3000 m

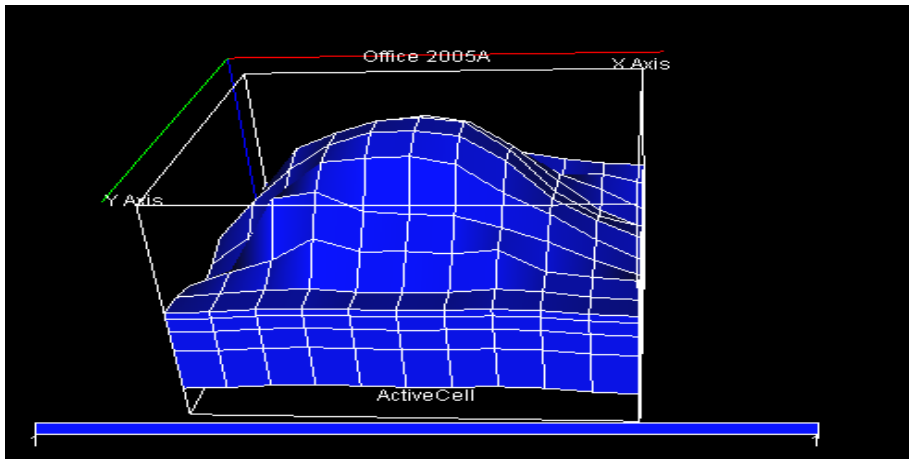


Fig.1: Regular XY grid.

The grid defines the dimensions and shape, including the petro-physical properties such as porosity, permeability, and the net-gross thickness. The figure above is the grid of an oil reservoir, 5000m by 5000m and 60m thick which consists of live oil and gas, with an aquifer of certain volume. The reservoir is subdivided into a 10× 10×4 grid. The grid defines the dimensions and shape, including the petro-physical properties such as porosity, permeability, and the net-gross thickness.

Table 2: Volumetric estimation of the three phase fluids

Region	Oil (SM3)	Water (SM3)	Gas (SM3)	Dis Gas (SM3)	Mobile Oil wrt Water (SM3)	Mobile Oil wrt Gas (SM3)
Field	51220873	1.7322041E+8	2.4575336E+10	2.4501866E+10	25002182	35571215

This Table 2 shows the amount of the fluid produced from the reservoir. This result shows clearly that the reservoir under study is producing more of gas.

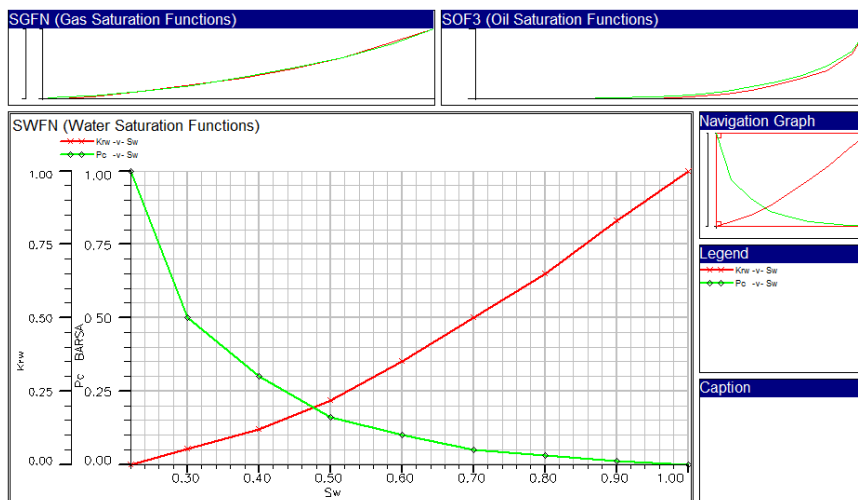


Fig.2: Graphical representation of the saturation functions.

The figure above is a plot of capillary pressure versus water saturation which shows that as the capillary pressure is decreasing the water saturation (wetting fluid) is increasing. This is an indication that the reservoir is water wet.

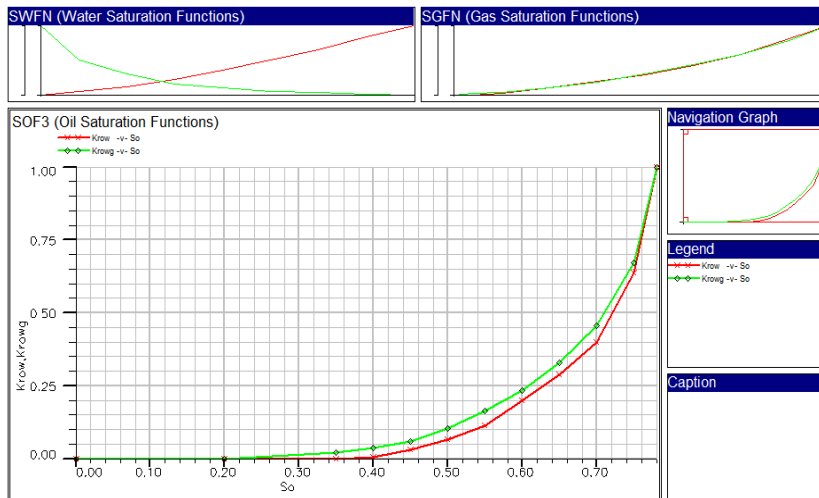


Fig.3: Graphical representation of the saturation functions.

The figure 3 above is a graph of relative permeability versus oil saturation for the modeled reservoir. The graph explains the relationship between the relative permeability and the oil saturation for a three phase system in which the reservoir is oil wet. The green line represents oil/water/gas permeability and the red line signifies oil/water permeability. It can be observe that at saturation of 50% the oil/water/gas permeability is equal to 0.1 which is greater than the oil/water permeability at the same saturation. This is as a result of the dissolved gas present in the oil which tends to make it less viscous thus enhancing mobility of oil in the three-phase more than the two-phase system.

Conclusion

The present study differs from other works that have studied on three phase fluid flow in the porous medium by the fact that it was able to relate the black oil model with the two basic volumetric estimation equations in order to determine the volume of oil, gas and water originally in place using Eclipse 100. It followed from the simulation results that the dynamic simulator was able to plot the saturation functions which were used to regulate the flow of oil, gas and water in the reservoir in terms of density variation, as well as the nature, shape and size of the reservoir. Also the simulation showed the volume of oil, gas and water to be $51220873m^3$, $1.722041E + 8m^3$ and $247533E + 10m^3$, respectively. These results showed that the reservoir is a gas reservoir since more gas is being produced.

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